

A new cohomological relation on $\overline{\mathcal{M}}_{2,14}$

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Main result

Theorem. There is a codimension-9 tautological relation on $\overline{\mathcal{M}}_{2,14}$ in cohomology which is **not generated by 3-spin relations**.

$$\underbrace{\mathcal{M}_{2,14}}_{\text{smooth}} \subset \underbrace{\mathcal{M}_{2,14}^{\text{ct}}}_{\text{compact type}} \subset \underbrace{\overline{\mathcal{M}}_{2,14}}_{\text{stable}}.$$

The two boundaries are

$$\Delta_0 = \overline{\mathcal{M}}_{2,14} \setminus \mathcal{M}_{2,14}^{\text{ct}}, \quad \Delta = \overline{\mathcal{M}}_{2,14} \setminus \mathcal{M}_{2,14}.$$

Δ_0 : curves with a nonseparating node. Δ : all nodal curves.

The cohomological source of the relation:

$$\text{Gr}_{16}^W H_c^{17}(\mathcal{M}_{2,14}^{\text{ct}}) \cong \mathbb{Q}(-8) \otimes \sigma_{(2^7)}.$$

The relation on Δ_0

$$\mathrm{Gr}_{16}^W H_c^{17}(\mathcal{M}_{2,14}^{\mathrm{ct}}) \cong \mathbb{Q}(-8) \otimes \sigma_{(27)}.$$

Compact-support localization gives

$$H^{16}(\overline{\mathcal{M}}_{2,14}) \longrightarrow H^{16}(\Delta_0) \longrightarrow H_c^{17}(\mathcal{M}_{2,14}^{\mathrm{ct}}) \longrightarrow H^{17}(\overline{\mathcal{M}}_{2,14}).$$

Taking the weight 16 piece and using exactness, we observe:

$0 \neq Q_0$ pushes forward to zero in $H_{16}(\overline{\mathcal{M}}_{2,14}) \cong H^{18}(\overline{\mathcal{M}}_{2,14})$.

$$f : \mathcal{M}_{2,14}^{\mathrm{ct}} \longrightarrow \mathcal{M}_2^{\mathrm{ct}} \cong \mathcal{A}_2, \quad \mathcal{M}_2^{\mathrm{ct}} \setminus \mathcal{M}_2 \cong \mathrm{Sym}^2 \mathcal{M}_{1,1}.$$

Decomposition theorem.

$$H_c^{17}(\mathcal{M}_{2,14}^{\mathrm{ct}}) \cong \bigoplus_q H_c^{17-q}(\mathcal{M}_2^{\mathrm{ct}}, R^q f_* \mathbb{Q}).$$

The $\mathrm{Sym}^2 \mathcal{M}_{1,1}$ part contributes nothing in weight 16. Only the following local-system piece contributes:

$$\mathrm{Gr}_{16}^W H_c^3(\mathcal{M}_2^{\mathrm{ct}}, \mathbb{V}_{7,7}) \cong \mathbb{Q}(-8).$$

Why the relation is not 3-spin

In type (2^7) , every 3-spin relation is a pushforward of a relation from a boundary stratum, hence already vanishes on $\Delta = \overline{\mathcal{M}}_{2,14} \setminus \mathcal{M}_{2,14}$.

It therefore suffices to show $Q \neq 0$ on Δ .

We reduce to proving surjectivity:

$$\mathrm{Gr}_{16}^W H_c^{17}(\mathcal{M}_{2,14}) \twoheadrightarrow \mathrm{Gr}_{16}^W H_c^{17}(\mathcal{M}_{2,14}^{\mathrm{ct}}).$$

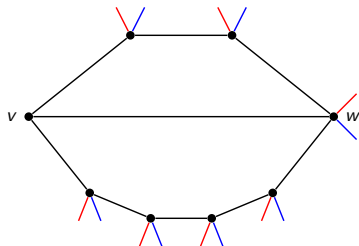
We compare two spectral sequences:

$$E_1^{p,q} = H_c^q(C_2^{14-p})^{\oplus c(14,14-p)} \implies H_c^{p+q}(\mathcal{M}_{2,14}),$$

$$\widetilde{E}_1^{p,q} = H_c^q(C^{14-p})^{\oplus c(14,14-p)} \implies H_c^{p+q}(F(C, 14)).$$

Comparing these spectral sequences helps us conclude surjectivity.

The explicit theta relation



Θ_{abc} has path lengths a, b, c with $a + b + c = 9$.

All eight vertices have genus 0.
Every vertex except v carries one red/blue pair.

Label pairs $(1, 8), \dots, (7, 14)$ and antisymmetrize the red and blue labels separately.

After: $[\Theta_{abc}]$ is a signed sum of labeled strata.

$$Q = [\Theta_{135}] - [\Theta_{144}] - [\Theta_{225}] + 2[\Theta_{234}] - [\Theta_{333}].$$

Theorem. $\text{cl}(Q) = 0 \in H^{18}(\overline{\mathcal{M}}_{2,14})$, but $Q \notin 3\text{Spin}_{2,14}^9$.

To read more: <https://people.math.ethz.ch/~rahul/Q-Notes.pdf>